Hybrid model for the spectral density of the time development of the excitation energy of Langmuir waves in unmagnetized plasmas

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(Received 5 April 2000)

A hybrid model has been developed for investigating the excitation of electron plasma waves in unmagnetized plasmas in the presence of a time-varying laser radiation field and a static Coulomb field. Energy spectral density of the excited plasma modes has been calculated analytically in wave number time space. The model also provides analytical expressions in a closed form for both the time-resolved and the time-averaged spectral density of the rate of energy absorption at resonance ($\omega_k \to \omega_0$).

PACS number(s): 52.25.−b, 52.35.−g, 52.50.−b, 52.20.Hv

I. INTRODUCTION

Understanding and modeling of the excitation of natural plasma modes via internal coupling mechanisms between different modes, nonlinear wave-particle interactions, and the coupling to an external driver, are of importance in plasma physical topics and related applications. Internal and external coupling processes are responsible for plasma heating, driving of plasma instabilities, and acceleration of charged particles to high energies. As an example, in inertial fusion targets induced scattering processes such as Raman and Brillouin instabilities are unwelcome since they prevent an effective plasma heating and energy deposition.

Although excited longitudinal plasma modes remain confined in the interaction region and will be dissipated in plasmas via nonlinear mechanisms and can contribute essentially to plasma heating, they can in some cases be of negative character. They can contribute to the production of very hot electrons of energies reaching values as large as 200 MeV [1]. These electrons can preheat the fusion core and accordingly prevent target compression to the necessary fusion temperatures.

In laser-matter interactions, parametric processes such as Raman, two-plasmon and Langmuir decay instabilities, all involve excited plasma modes [2–4]. In magnetic fusion applications, plasma modes can be driven resonantly via relativistic electron or ion beams [5–8]. Electron plasma waves can contribute via nonlinear resonant interaction with particles or waves to the generation of charged particles accelerated to high energy that are trapped in the wake field of the wave or to the excitation of ion sound waves via the electromagnetic decay instability [9–13]. Energy dissipation of the excited waves and their interaction with the magnetic field are of importance for plasma heating and plasma diagnostics [9,14,15].

Growth rates and conditions for excitation of longitudinal Langmuir waves via a variety of drivers such as the beating of two laser beams [16–20], a sequence of laser pulses [21], nonmonochromatic microwave beams [22], mass oscillations of relativistic electron in an external electric field, and dynamically nonlinear electrons interacting with a beam of fast positive ions [5], have been considered extensively in previous works. In the present paper, attention will in turn be devoted to the problem of exciting electron plasma waves in the presence of a static Coulomb field and time-varying laser radiation field.

In Sec. II, the ballistic oscillator model will be introduced and used in the following sections for studying the excitation of longitudinal plasma waves. In Sec. III, the model equations will be solved for electron deflections driven by a static ion Coulomb field. In Sec. IV, electron deflections driven by both a static Coulomb field and an oscillating radiation field will be determined, and the resonant and nonresonant spectral densities of the excitation energy of Langmuir waves will be calculated. Finally in Sec. V, conclusions will be presented.

II. OSCILLATOR MODEL

Assuming a Lorenz gas of electrons and an immobile neutralizing background of positive ions with uniform density $Zn_{0i}=n_{0e}$, where $n_{0\mathrm{e}}$ is the equilibrium density of electrons, and upon linearizing the equations of the electron fluid,

$$\frac{\partial n_e}{\partial t}+\vec{\nabla}\cdot(n_e\vec{u}_e)=0, \tag{1}$$

$$\frac{\partial \vec{u}_e}{\partial t}+(\vec{u}_e\cdot\vec{\nabla})\vec{u}_e=-\frac{\vec{\nabla}p_e}{mn_e}-\frac{e}{m_e}(\vec{E}+\vec{u}_e\times\vec{B}), \tag{2}$$

$$\vec{\nabla}\cdot\vec{E}=\frac{\rho_{0i}+\rho_e}{\epsilon_0}, \tag{3}$$

one obtains the following equation for the deflection of an electron during its flight on a stationary scattering center of a positive charge Ze:

$$\frac{\partial^2\vec{\delta}(\vec{r},t)}{\partial t^2}-\frac{3}{2}v_{\mathrm{th}}^2\nabla^2\vec{\delta}(\vec{r},t)+\omega_{\mathrm{pe}}^2\vec{\delta}(\vec{r},t)=-\frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{\vec{r}}{|\vec{r}|^3}, \tag{4}$$

where $\vec{\delta}(\vec{r},t)$ is the fluctuation of an element of the Lorenz gas of electrons and, as such, also represents the displace-

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ment or deviation of the single electrons from the equilibrium position, and ω_{pe} is the electron plasma frequency. Here υ_{th} is the electron thermal speed and $|\vec{r}|$ is the relative separation of an electron-ion pair.

For cold plasmas ($\upsilon_{\mathrm{th}}=0$), Eq. (4) was derived by Mulser *et al.* and was used for determination of the stopping power of heavy ions in plasmas and of the electron-ion collision frequency in intense laser fields [23,24]. It is known in the literature as the oscillator model. We notice that Eq. (4) includes the spatial variations of the electron deflection and will therefore account for the excitation of plasma waves, and for energy transfer from the incident pump wave into plasma modes and particles. It includes also both the collective and noncollective dynamics of electrons that are bound to the ion scattering centers.

In arriving at Eq. (4) relativistic effects are ignored. This is valid for small electron thermal and oscillation energies and a small photon energy $\hbar\omega_0$ as compared to the electron rest energy.

Let $\vec{\delta}$ be composed of one component parallel to $\vec{r}(\vec{\delta}_\parallel)$ and a perpendicular one $\vec{\delta}_\perp$, then Eq. (4) splits into the following equations,

$$\frac{\partial^2\delta_\perp}{\partial t^2}-\frac{3}{2}\upsilon_{\mathrm{th}}^2\vec{\nabla}^2\delta_\perp+\omega_{\mathrm{pe}}^2\delta_\perp=0, \tag{5}$$

$$\frac{\partial^2\delta_\parallel}{\partial t^2}-\frac{3}{2}\upsilon_{\mathrm{th}}^2\vec{\nabla}^2\delta_\parallel+\omega_{\mathrm{pe}}^2\delta_\parallel=-\frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{1}{|\vec{r}|^2}. \tag{6}$$

Fourier transforming Eq. (5) in space gives

$$\frac{d^2\delta_{\perp,k}(t)}{dt^2}+\left(\omega_{\mathrm{pe}}^2+\frac{3}{2}\upsilon_{\mathrm{th}}^2k^2\right)\delta_{\perp,k}(t)=0, \tag{7}$$

$$\delta_{\perp,k}(t)=A\sin\omega_k t+B\cos\omega_k t, \tag{8}$$

$$\omega_k^2=\omega_{\mathrm{pe}}^2+\frac{3}{2}\upsilon_{\mathrm{th}}^2k^2, \tag{9}$$

where ω_k in Eq. (9) represents the dispersion relation for a free propagating electron plasma wave corresponding to the collective motion of electrons in the direction perpendicular to the axis connecting the electron-ion pair. Upon applying the initial conditions $\delta_{\perp,k}(0)=0$ and $\dot{\delta}_{\perp,k}(0)=0$, the unique solution of Eq. (7) becomes $\delta_{\perp,k}(t)=0$. Along the relative electron-ion separation, Eq. (6) for $\delta_\parallel$ describes electron plasma waves that are bound to the scattering center and that might be driven resonantly by the driver field on its right-hand side.

III. STATIC DRIVER

In this section, Eq. (6) will be solved for the electron displacement $\delta_{\parallel,k}(t)$ in a static Coulomb field. The physical situation in Eq. (6) is more interesting than in Eq. (5) due to the presence of a driver field on its right-hand side. Collective motion of electrons along the relative electron-scattering center separation can be driven by the field resonantly when certain coupling conditions are satisfied.

Fourier transforming Eq. (6) in space only results in

$$\frac{d^2\delta_{\parallel,k}(t)}{dt^2}+\left(\omega_{\mathrm{pe}}^2+\frac{3}{2}\upsilon_{\mathrm{th}}^2k^2\right)\delta_{\parallel,k}(t)=\frac{iZe^2}{\epsilon_0 m_e k} \tag{10}$$

Equation (10) describes in k-t space an electron plasma wave harmonically bound to the static ion Coulomb field. Using ω_k introduced in Eq. (9), the general solution of Eq. (10) becomes,

$$\delta_{\parallel,k}(t)=C\sin\omega_k t+D\cos\omega_k t+\frac{iZe^2}{\epsilon_0 m_e k\omega_k^2}. \tag{11}$$

Upon applying the initial conditions $\delta_{\parallel,k}(0)=0$ and $\dot{\delta}_{\parallel,k}(0)=0$, the general solution in Eq. (11) becomes

$$\delta_{\parallel,k}(t)=\frac{2iZe^2}{\epsilon_0 m_e}\frac{\sin^2(\omega_k t/2)}{k\omega_k^2} \tag{12}$$

The total-energy spectral density $\epsilon_k(t)$ corresponding to the solution in Eq. (12) is given by

$$\begin{aligned}\epsilon_k(t)&=\frac{m}{2}\dot{\delta}_k(t)\dot{\delta}_k^*(t)+\frac{m}{2}\omega_k^2\delta_k(t)\delta_k^*(t)\\&=2m\left(\frac{Ze^2}{m\epsilon_0}\right)^2\frac{\sin^2(\omega_k t/2)}{k^2\omega_k^2}=\epsilon_k(0)\sin^2\frac{\omega_k t}{2}.\end{aligned} \tag{13}$$

The spectral density of the energy absorption rate $\dot{\epsilon}_k(t)$ is

$$\frac{d\epsilon_k(t)}{dt}=m\left(\frac{Ze^2}{m\epsilon_0}\right)^2\frac{\sin\omega_k t}{k^2\omega_k}=\frac{\omega_k}{2}\epsilon_k(0)\sin\omega_k t \tag{14}$$

$$=\omega_k\cot\frac{\omega_k t}{2}\epsilon_k(t)\equiv\omega_k(t)\epsilon_k(t), \tag{15}$$

where $\omega_k(t)$ gives the rate at which the spectral energy changes with time and ω_k is defined in Eq. (9),

$$\omega_k(t)=\omega_k\cot\frac{\omega_k t}{2} \tag{16}$$

The time-averaged spectral density of the energy absorption rate over one oscillation $\tau_k=\omega_k^{-1}$ of the kth plasma mode is zero ($\langle\dot{\epsilon}_k(t)\rangle_{\mathrm{cycle}}$) since a static Coulomb field does not result in an energy transfer.

As pointed out by Rand [25], the complexity of the electron motion can be reduced by transforming to an oscillating coordinate system, so that it appears as if the the ions are oscillating and the electrons are unaffected by the radiation field. Therefore, the ions produce a time-dependent field, in which the electrons scatter. The radiation field uses the oscillating ions as an intermediary to transfer energy to the electrons and to drive the natural plasma modes resonantly. This case will be investigated in Sec. IV.

IV. TIME VARYING DRIVER

In this section, Eq. (6) will be solved for the electron deviation $\delta_{\parallel}\equiv\delta$ in the presence of an external time-varying radiation field and a static Coulomb field. The ion field on the right-hand side. of Eq. (6) will play an intermediary role in transferring the light pressure from the radiation field to the electrons. This is essentially a three-body problem by which the electrons can be accelerated or decelerated in the ion field leading to absorption (inverse bremsstrahlung) or emission (bremsstrahlung) of radiations.

It is of importance to account for the coupling between the laser field and the scattering center, which in turn will transmit the ponderomotive force to the electrons. Assuming a harmonic laser field of the form $\vec{E}(t)=\vec{E}_0\sin\omega_0 t$, and upon transforming to a frame of reference that oscillates with the electrons, so that the ions seem to oscillate in the laser field relative to the electrons, Eq. (6) becomes

$$\frac{\partial^2\delta(\vec{\rho},t)}{\partial t^2}-\frac{3}{2}v_{\rm th}^2\vec{\nabla}_\rho^2\delta(\vec{\rho},t)+\omega_{\rm pe}^2\delta(\vec{\rho},t)=-\frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{1}{|\vec{\rho}+\vec{y}_{\rm osc}\cos\omega_0 t|^2},\quad (17)$$

where $\vec{\rho}$ denotes the electron position with respect to the ion in the oscillating frame and $\vec{y}_{\rm osc}=e\vec{E}_0/m_e\omega_0^2$ is the oscillation amplitude of the electron in the laser field. Fourier transforming Eq. (17) in space gives,

$$\frac{d^2\delta_{\vec{k}}(t)}{dt^2}+\left(\omega_{\rm pe}^2+\frac{3}{2}v_{\rm th}^2k^2\right)\delta_{\vec{k}}(t)=\frac{iZe^2}{\epsilon_0 m_e k}e^{i\vec{k}\cdot\vec{y}_{\rm osc}\cos\omega_0 t}.\quad (18)$$

Using

$$e^{i\vec{k}\cdot\vec{y}_{\rm osc}\cos\omega_0 t}=\sum_{l=-\infty}^{\infty}i^l\ e^{il\omega_0 t}J_l(\vec{k}\cdot\vec{y}_{\rm osc}),$$

Eq. (18) becomes

$$\frac{d^2\delta_{\vec{k}}(t)}{dt^2}+\omega_k^2\delta_{\vec{k}}(t)=\sum_{l=-\infty}^{\infty}g_l(k,y_{\rm osc})e^{il\omega_0 t},\quad (19)$$

$$g_l(k,y_{\rm osc})=\frac{i^{l+1}Ze^2}{\epsilon_0 m_e k}J_l(\vec{k}\cdot\vec{y}_{\rm osc}).\quad (20)$$

The general solution of Eq. (19) is

$$\delta_{\vec{k}}(t)=A\sin\omega_k t+B\cos\omega_k t+\sum_{l=-\infty}^{\infty}\frac{g_l(k,y_{\rm osc})}{\omega_k}(\sin\omega_k t\,I_1-\cos\omega_k t I_2),\quad (21)$$

$$I_1=\int^t d\tau\cos\omega_k\tau e^{il\omega_0\tau},\quad I_2=\int^t d\tau\sin\omega_k\tau e^{il\omega_0\tau}\quad (22)$$

Upon evaluating the indefinite integrals I_1 and I_2, Eq. (21) becomes

$$\delta_{\vec{k}}(t)=A\sin\omega_k t+B\cos\omega_k t+\sum_{l=-\infty}^{\infty}g_l(k,y_{\rm osc})\frac{e^{il\omega_0 t}}{\omega_k^2-l^2\omega_0^2}.\quad (23)$$

Applying the initial conditions $\delta_k(0)=0$ and $\dot{\delta}(0)=0$, Eq. (23) reduces into

$$\begin{aligned}\delta_{\vec{k}}(t)&=\sum_{l=-\infty}^{\infty}\frac{g_l(k,y_{\rm osc})}{\omega_k^2-l^2\omega_0^2}\left(e^{il\omega_0 t}-\cos\omega_k t-\frac{il\omega_0}{\omega_k}\sin\omega_k t\right)\\&=\frac{2iZe^2}{\epsilon_0 m_e}\frac{\sin^2(\omega_k t/2)}{k\omega_k^2}+\sum_{l=1}^{\infty}\frac{\cos\omega_k t}{\omega_k^2-l^2\omega_0^2}[g_l(k,y_{\rm osc})\\&\quad+g_{-l}(k,y_{\rm osc})]+\sum_{l=1}^{\infty}\frac{1}{\omega_k^2-l^2\omega_0^2}[g_l(k,y_{\rm osc})e^{il\omega_0 t}\\&\quad+g_{-l}(k,y_{\rm osc})e^{-il\omega_0 t}],\end{aligned}\quad (24)$$

where the first term on the right-hand-side of Eq. (24) gives the contribution of the static Coulomb field to the total electron deflection ($l=0$) and the two terms under the sum represent the contribution of the laser harmonics ($l\geqslant 1$).

The total-energy spectral density $\epsilon_k(t)$ corresponding to the electron deflection in Eq. (24) is

$$\begin{aligned}\epsilon_k(t)&=\frac{m}{2}\dot{\delta}_k(t)\dot{\delta}_k^*(t)+\frac{m}{2}\omega_k^2\delta_k(t)\delta_k^*(t)\\&=2m\left(\frac{Ze^2}{m\epsilon_0}\right)^2\frac{\sin^2(\omega_k t/2)}{k^2\omega_k^2}\\&\quad+\sum_{l=1}^{\infty}\left(\frac{Ze}{\epsilon_0 m}\right)^2\frac{2mJ_l^2}{k^2(\omega_k^2-l^2\omega_0^2)^2}[(\omega_k^2+l^2\omega_0^2)(1\\&\quad-\cos l\omega_0 t\cos\omega_k t)-2l\omega_0\omega_k\sin l\,\omega_0 t\sin\omega_k t].\end{aligned}\quad (25)$$

The first term on the right-hand-side of Eq. (25) corresponds to the case $l=0$ and gives the contribution of the static Coulomb field to the total-energy spectral density of the kth longitudinal plasma mode. This corresponds to a purely static driver case discussed in Sec. III. The term under the sum on the right-hand-side. of Eq. (25) corresponds to the contribution of all laser harmonics $l\geqslant 1$ to the energy spectral density of the kth plasma mode.

The time rate of change of the energy spectral density $\epsilon_k(t)$ is

$$\begin{aligned}\frac{d\epsilon_k(t)}{dt}&=m\left(\frac{Ze^2}{m\epsilon_0}\right)^2\frac{\sin\omega_k t}{k^2\omega_k}+\sum_{l=1}^{\infty}\left(\frac{Ze^2}{\epsilon_0 m}\right)^2\frac{2mJ_l^2}{k^2(\omega_k^2-l^2\omega_0^2)}\\&\quad\times[\omega_k\cos l\omega_0 t\sin\omega_k t-l\omega_0\sin l\omega_0 t\cos\omega_k t]\end{aligned}\quad (26)$$

Near resonance when $\omega_k\rightarrow l\omega_0$, Eq. (26) becomes

$$\left(\frac{d\Delta\epsilon_k(t)}{dt}\right)_{\text{Res.}}=m\left(\frac{Ze^2}{m\epsilon_0}\right)^2\frac{\sin l\,\omega_0 t}{k^2 l\omega_0}+\sum_{l=1}^{\infty}\left(\frac{Ze^2}{\epsilon_0 m}\right)^2\frac{mJ_l^2}{k^2 l\omega_0}\left[l\omega_0 t+\frac{1}{2}\sin 2l\omega_0 t\right] \tag{27}$$

Equation (27) gives an analytical expression for the time-varying spectral density of the energy absorption rate when the kth plasma mode is driven resonantly. Time averaging of Eq. (27) over one laser oscillation gives

$$\left\langle\left(\frac{d\Delta\epsilon_k(t)}{dt}\right)_{\text{Res.}}\right\rangle_{\text{cycle}}=\sum_{l=1}^{\infty}\left(\frac{Ze^2}{\epsilon_0 m}\right)^2\frac{m\pi J_l^2}{k^2 l\omega_0}. \tag{28}$$

The energy transfer at resonance ($\omega_k\rightarrow\omega_0$) in Eq. (28) is finite and free of singularities. The contribution of the static Coulomb field over a laser period to the total spectral energy absorption rate vanishes and only the contribution of laser harmonics $l\geqslant 1$ remains.

V. CONCLUSIONS

In this paper, a hybrid model based on the ballistic oscillator equation [Eq. (4)] has been developed for studying the excitation of electron plasma waves in unmagnetized plasmas. The model accounts for both a static Coulomb and time-varying laser radiation fields. It provides analytical expressions for the spectral density of the excitation energy $\epsilon_k(t)$ of electron plasma waves, and the corresponding time-resolved and time-averaged energy absorption rates $\dot{\epsilon}_k(t)$ when the plasma modes are driven by the laser harmonics resonantly. To the best of the knowledge of the author, there are no previous works that might be used to compare with the analytical results presented.

The case of electron scattering driven only by a static Coulomb field has been discussed in Sec. III. The rate of change of the spectral density of the excitation energy $\dot{\epsilon}_k(t)$ is found to vary nonlinearly with time with a rate of change $\omega_k(t)=\omega_k(0)\cot(\omega_k t/2)$. The time averaged energy absorption rate $\dot{\epsilon}_k(t)$ over a plasma wave period $\tau_k=\omega_k^{-1}$ vanishes, a result that means a static Coulomb field results in no net energy transfer to the plasma electrons.

In the presence of both a time-varying laser radiation and a static Coulomb fields, the complexity of the electronic motion has been reduced by transforming to an oscillating coordinate system. In the oscillating frame the ions are oscillating and the electrons are unaffected by the radiation field. The ions will therefore produce a time-varying Coulomb field, on which the electrons scatter.

In Sec. IV, Eq. (4) has been solved as an initial value problem in wavenumber-time space for the electron deviation. The total spectral density of the excitation energy and of the energy absorption rate $\dot{\epsilon}_k(t)$ have been calculated. Contributions of both the Coulomb and the time-varying radiation field [Eq. (26)] have been determined.

Of interest is the determination of $\dot{\epsilon}_k(t)$ at resonance [as $\omega_k\rightarrow\omega_0$]. Equation (27) gives an analytical expression for the time variations of the spectral density of the energy absorption rate when the kth plasma mode is driven resonantly. The energy transfer for $\omega_k\rightarrow\omega_0$ is found to have a finite value and is free of singularities. Upon averaging over a laser period the contribution of the static Coulomb field to the total spectral energy absorption rate vanishes and only the contribution of laser harmonics $l\geqslant 1$ remains [Eq. (28)].

ACKNOWLEDGMENTS

The author would like to thank Professor Dr. P. Mulser from the TU - Darmstadt, IAP, TQE, Germany, for his fruitful comments on the oscillator model and for the financial support for his stay there in 1999.

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